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Interband Faraday effect in semiconductors*

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Using the theory developed by Roth, the theoretical values of the interband Faraday rotation for Ge, GaAs, GaSb, InSb, and InAs are calculated from a knowledge of the previously determined band edge parameters The rotation near the band edge singularity and near the low frequency limit are calculated Two variable parameters, which are the measure of the goodness of fit of the theory with experiment are employed, for getting the overall picture from the limiting values of the rotation, as the exact calculation on the basis of the present theory is extremely difficult. After fitting the parameters with experiment the theoretical calculations agree reasonably well with experimental observations

1. Introduction

Since the end of the fifty's there has been a considerable interest in the Faraday effect in semiconductors for the information it can provide about the band parameters The main problem in the interband Faraday rotation spectrum near the Γ point energy gap at the present moment is the explanation of the positive rotation near the low frequency region and the change in sign of rotation as the gap is approached as is found in Ge (Hartmann & Kleman 1960, Walton & Moss 1961), GaAs (Cardona 1961, Piller 1964) and GaSb (Piller & Patton 1963) In InSb and InAs the rotation is negative and large (Smith, Pidgeon & Prosser 1962, Cardona 1961) InP, GaP, AlSb, CdS (Kimmel 1957; Moss, Walton & Ellis 1962, Balkanski & Hopfield 1962) and Si (Piller & Potter 1962) all have positive rotations Here we are primarily concerned with the first two groups of materials which have Ge-like band edges at the Γ point Of the third group, Si has a different conduction band edge at the Γ point, and the band parameters of the other materials are not well known The behaviour of the Faraday rotation of the three groups appear to be related to the differences in the magnitude of the band gaps. The materials of the third group have large band gaps (1.34–3 6) eV The materials of the second group have small gaps (0 23–0 41) eV, while the first group have intermediate band gaps (0 82–1 53) eV

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The relation between the angle θ of rotation of the plane of polarization per unit length and the conductivity tensor $\sigma_{\alpha\beta}$ for the magnetic field in the Z-direction, according to macroscopic theory (Stephen & Lidiard 1959) is

$$\theta = \frac{2\pi\sigma_{r\,xy}}{nc} \qquad \ldots \quad (1.1)$$

where the index r denotes the real part and n is the refractive index. Using the methods of semiclassical radiation theory, it is shown by Bennett & Stern (1960) that the conductivity tensor may be written as

$$\sigma_{\alpha\beta} = \frac{e^2}{im\omega} \sum_{nn'} \left\{ \delta_{\alpha\beta}\, \delta_{nn'} f_n + \frac{1}{m} \frac{P^{\beta}_{nn'} P^{\alpha}_{n'n} f_{nn'}}{E_{nn'} + \omega} \right\} \qquad \ldots \quad (1.2)$$

where P is the kinetic momentum operator, n and n' denote the eigenstates of the one electron Bloch Hamiltonian in the presence of the magnetic field, f_n is the Fermi distribution function for energy E_n, $f_{nn'} = f_n - f_{n'}$ and $E_{nn'} = E_n - E_{n'}$, α and β are the vector components.

Roth (1964, hereafter referred to as R) using the modified Bloch representation (Roth 1962) in Eq (1.2), written in the form of a trace, obtained an expression for the conductivity tensor to the first order in magnetic field due to direct interband transitions. Her result is

$$\sigma_{xy}^{(1b)} = -ie^2\hbar\epsilon_{\alpha\beta}\,\epsilon_{\gamma\delta} \sum_{nn'n''k}{}' \Big[\Pi^{\alpha}_{nn'}(x^{\gamma}(\Pi^{\delta} + v^{\delta}) + S_{\gamma\delta})_{n'n''} \Pi^{\beta}_{n''n}$$

$$-(\Pi^{\alpha}x^{\gamma} + i\delta_{\alpha\gamma}/m)_{nn'}(\Pi^{\delta} + v^{\delta})_{n'n} \Pi^{\beta}_{n''n} \Big] \frac{1}{E_{n'n''}}$$

$$\times \left\{ \frac{f_{nn'}}{E^2_{nn'} - \omega^2} - \frac{f_{nn''}}{E^2_{nn''} - \omega^2} \right\} \qquad \ldots \quad (1.3)$$

where the prime on the sum excludes the case where the three energies are equal. This is the most general result of interband Faraday rotation. The symbols having the same meanings as in R. $\sigma_{xy}^{(1b)}$ consists essentially of two terms, the first is the contribution from Zeeman interaction H' of a band electron and the second is the contribution from the change in matrix element Π' due to the presence of the magnetic field.

Roth (1964) used eq (1.3) to obtain interband Faraday rotation for semiconductors having a s-like conduction band and spin-orbit split p-type valence band as is found in Ge and some III–V compounds, with the following assumptions :

(i) The evaluation of the matrix elements at the Γ point band edge.

(ii) The consideration of the terms $n' = n''$ only, which we call Zeeman term and the assumption that the other terms are negligible.

(iii) The consideration of the contribution of the transitions from the doubly degenerate valence bands V_1 and V_2 to the conduction band, and the assumption that the contribution of the transitions from the split off valence band V_3 to conduction band are negligible

The result obtained with these assumptions is

$$\theta_1 = \theta_{01} H F_2(\omega/E_g) \quad \ldots \quad (1.4)$$

where

$$\theta_{01} = (5\ 17\ f/n\sqrt{E_g})[(\mu_h/m)^{3/2} + (\mu_1/m)^{3/2}]g_{eff} \quad \ldots \quad (1.5)$$

in deg/cm-KG, and E in eV. Again the other symbols have the same meanings as in I. The function $F_2(\omega/E_g)$ is real below the gap and complex above the gap. When $F_2(\omega/E)$ is complex we have to take its real part to calculate the rotation. The form of the real part of the function has a dispersion like character near the gap although unsymmetrical.

Application of eq. (1.3) to a pair of simple spherical bands, one of which, the valence band, is occupied and the other, the conduction band is empty, evaluating the matrix element at the band edge and using the approximation (ii) above gives

$$\theta = \frac{e^2 l_0}{4nc(2m\bar{E}_g)^{\frac{1}{2}}} (\mu/m)^{3/2}(g_c g_c' - g_v g_v') F_2(\omega/E_g) \quad (1.6)$$

where the Zeeman interaction is written in terms of the g-factor and g' represents the contribution to g-factor from one band only.

Eqs (1.4) and (1.5) enable Roth (1964) to predict from the knowledge of the band edge parameters the sign and the order of magnitude of Faraday rotation near the singularity at the Γ point correctly for the materials we propose to study. For the low frequency region, however, her calculations give the right frequency dependence for Ge, GaAs, and GaSb, but do not give the correct magnitude or sign. Thus the next problem which needs investigation is to account for the positive rotation near the low frequency region for Ge, GaAs, and GaSb. It was suggested by Boswarva & Lidiard (1962) and Pidgeon, Summer, Arai & Smith (1964) that the contribution to the Faraday rotation due to the L point transition is responsible for the positive rotation near the low frequency region for Ge. We have calculated the contribution to the Faraday rotation due to the L point transition (to be reported elsewhere). Results of our calculations for Ge show that there is no positive contribution near the low frequency region due to L point transition. We shall therefore propose an alternative explanation for the positive rotation at long wavelengths, namely that it is due to transitions throughout the entire Brillouin zone and not due to a few band edges.

In section 2, we will include the Zeeman term for the transitions from the split off valence band V_3 to the conduction band. In section 3, we will consider the contribution to the rotation from the change in matrix element. In section 4, we will try to compensate the single band gap approximation (i) by including other band gaps in terms of an effective band gap. We will compare the total contribution with the experimental results of materials of the first two groups mentioned above in section 5. Finally, section 6 contains discussion and concluding remarks.

2 Contribution to Rotation Due to Zeeman Interaction

For a pair of simple spherical bands for $n' = n''$ only the Zeeman term in eqr. (1.3) contributes. This is why we call this term as Zeeman term, although in the case of real semiconductors, the second term in eq. (1.3) contributes, which is due to the degeneracy of the valence bands V_1 and V_2. The contribution to the Faraday rotation θ_1 from the Zeeman term due to the transition from V_1 and V_2 to the conduction band, is given in eq. (1.4).

If we let θ_2 be the contribution to the Faraday rotation from the Zeeman term due to transition from the split off valence band V_3 to the conduction band, then using relation (1.6) we can write θ_2 in the form

$$\theta_2 = \theta_{02} H F_2(\omega/E_g, \Delta) \tag{2.1}$$

where

$$\theta_{02} = (6 \times 5.17/n(E_g + \Delta)^{\frac{1}{2}})(\mu_3/m)^{3/2}(g_c g_c' - g_3 g_3')$$

in deg/cm-kG, with E_g and Δ, the spin-orbit splitting in eV, and

$$g_c = 2 - f[2\Delta/3(E_g + \Delta)] \tag{2.3}$$

$$g_c' = (2/3)f'; \; f' = \left(\frac{E_g}{E_g + \Delta}\right) f \tag{2.4}$$

$$g_3 = 2 - (4/3)(3\kappa' + 1) \tag{2.5}$$

$$g_3' = -(2/3)f'. \tag{2.6}$$

The subscripts c and 3 refer to the condition band and the split off valence band V_3 respectively. κ' is obtained from κ by replacing f by f'. μ_3 is the reduced effective mass obtained from the conduction band effective mass m and the split off valence band effective mass m_{v_3}.

3 Contribution to the Rotation Due to the Change of Matrix Element

In eqs. (1.3) there are three possible types of terms for three possible arrangements of the band indices n, n', and n'' among conduction and valence bands.

The terms with $n' \cong n''$, $f_{n'} \cong f_{n''}$, which we call the Zeeman term has already been considered in the last section. The next possible terms are with $n \cong n'$, $f_n \cong f_{n'}$ The matrix elements in this term vanished at $k = 0$, and therefore we shall neglect contribution from this term Finally, we have the terms with $n \cong n'$, $f_n \cong f_{n'}$ such that n and n' go together either to the conduction band or to the valence band Only the second term in eq (1 3) contributes to such terms and hence we call this as the contribution due to the change in matrix element For this case eq (1 3) reduces to

$$\sigma_{xy}^{(ab)} = -ie^2\hbar\epsilon_{\alpha\beta}\,\epsilon_{\gamma\delta}\sum_{i\,j\,n''k}{}' \left[\left\{\Pi^{\alpha}x^{\gamma} + i\,\frac{\delta_{\alpha\gamma}}{m}\right\}(\Pi^{\delta}+r^{\delta})_{jn''}\,\Pi^{\beta}_{n''i}\right.$$

$$\left.\times \frac{f_{in''}}{E_{jn''}(E_{in''}{}^2-\omega^2)}\right] \qquad \ldots \quad (3\,1)$$

where i, j is a set of degenerate bands not containing n'' and the prime on the sum exclude the case $i = j = n''$.

Now we are interested in real semiconductors having s-like conduction band and spin orbit split p-type valence band as is found in Ge and some III–V compounds. Assuming the convention that when i and j refer to a particular set of degenerate bands, the band n'' does not belong to that degenerate set, and evaluating the matrix elements at the band edge eq. (3 1) may be written as

$$\sigma_{xy}^{(a\,b)} = e^2\hbar\epsilon_{\alpha\beta}\,\epsilon_{\gamma\delta}\sum_{i\,j\,n''k}\left[\left(\sum_{p}\frac{\Pi_{ip}^{\alpha}\,\Pi_{pj}^{\gamma}}{E_{op}} + \frac{\delta_{\alpha\gamma}\,\delta_{ij}}{m}\right)\frac{\Pi^{\delta}_{jn''}\,\Pi^{\beta}_{n''i}}{E_{on''}}\right]_{k=0} \times$$

$$\times \frac{f_{in''}}{(E^2_{in''}-\omega^2)} \qquad \ldots \quad (3\,2)$$

where E_0 denotes the energy of the degenerate set at the band edge Using the effective mass matrix elements $D_{ij}{}^{\alpha\beta}$ as introduced by Luttinger & Kohn (1955),

$$D_{ij}{}^{\alpha\beta} = \frac{\delta_{\alpha\beta}\,\delta_{ij}}{2m} + \sum_{p}\frac{\Pi^{\alpha}_{ip}\,\Pi^{\beta}_{pj}}{E_{op}} \qquad \ldots \quad (3\,3)$$

and denoting contribution to $D_{ij}{}^{\alpha\beta}$ from one band only by

$$D'_{ij}{}^{\alpha\beta}\,(n'') = \frac{\delta_{\alpha\beta}\,\delta_{ij}}{2m} + \frac{\Pi^{\alpha}_{in''}\,\Pi^{\beta}_{n''j}}{E_{on''}} \qquad \ldots \quad (3\,4)$$

and taking the magnetic field H in the Z direction we get,

$$\sigma_{xy}^{(a,b)} = e^2\hbar \sum_{ijn''k} \Big[\Big(D_{ij}^{xz} + \frac{\delta_{ij}}{2m}\Big)\Big(D'_{jl}{}^{yy}(n'') - \frac{\delta_{ij}}{2m}\Big) + \Big(D_{ij}^{yy} + \frac{\delta_{ij}}{2m}\Big)\Big(D'_{jl}{}^{xz}(n'') - \frac{\delta_{ij}}{2m}\Big) - D_{ij}^{xy} D'_{jl}{}^{yz}(n'') - D_{ij}^{yz} D'_{jl}{}^{xy}(n'')\Big]_{k=0} \frac{f_{in''}}{(\bar{E}^2_{in''} - \omega^2)} \quad \ldots (3.5)$$

Let us first consider the contribution due to the transition from the degenerate lence bands V_1 and V_2 to the conduction band. The basis functions corresponding to the valence band V_1 are,

$$\phi_1 = (i/\sqrt{2})(X + iY)\alpha$$

$$\phi_1' = (i/\sqrt{2})(X - iY)\beta$$

corresponding to the valence band V_2 are,

$$\phi_2 = (i/\sqrt{6})[(X + iY)\beta - 2Z\alpha]$$

$$\phi_2' = (i/\sqrt{6})[-(X - iY)\alpha - 2Z\beta]$$

corresponding to the condition band c are

$$\phi_c = S\alpha$$

$$\phi_c' = S\beta$$

The subscripts i, j in the effective mass when denoting the valence band may refer to any of the basis ϕ_1, ϕ_1', and ϕ_2, ϕ_2', and n'' the basis ϕ_c, $\phi_{c'}$ When the subscripts i, j denote the conduction band may refer to the basis $\phi_c, \phi_{c'}$, and n'' to ϕ_1, ϕ_1' and ϕ_2, ϕ_2' To work in a representation in which the Hamiltonian is diagonal we replace X by $\boldsymbol{R}\,\boldsymbol{s}$, Y by $\boldsymbol{R}.\boldsymbol{t}$, and Z by $\boldsymbol{R}\,\boldsymbol{r}$ in the old representation where

$$R = Xi + Yj + Zk$$

and $\boldsymbol{r} = \boldsymbol{k}/k$, with $\boldsymbol{s}$ and $\boldsymbol{t}$ unit vectors perpendicular to $\boldsymbol{r}$ and to each other, and assume the spins to be quantized in the $\boldsymbol{r}$ direction

Using the new representation we calculate $D_{ij}^{\alpha\beta}$. When $i, j = c, c'$ we consider only the contribution from the valence bands V_1, V_2, and V_3 but when $i, j = V_1$, V_2, V_3 all the bands have been considered, as in R With these matrix elements we evaluate the sum of the expression in square bracket of eq (3.5) at the band edge The sum over k in eq (3.5) gives

$$\sum_k \frac{1}{\bar{E}_{cv}^2 - \omega^2} = \frac{(\mu)^{3/2}}{2\pi(2\bar{E}_g)^{\frac{1}{2}}} \cdot \frac{1}{x}\Big[\sqrt{1+x} - \sqrt{1-x}\Big]$$

where μ is the reduced effective mass and $x = \omega/E_g$. As in R, to obtain the correct low frequency dependence, we subtract from the above sum the $\omega = 0$ term. If we let θ_3 be the contributions to Faraday rotation due to the change in matrix element for transitions from the degenerate valence bands V_1 and V_2 to the conduction band, then we get

$$\theta_3 = \theta_{03} H f(\omega/E_g) \qquad \ldots \quad (3\,6)$$

where,

$$f(x) = \frac{1}{x}\left[\sqrt{1+x}-\sqrt{1-x}\right]-1 \qquad \ldots \quad (3.7)$$

and,

$$\theta_{03} = -(5\,17\, f/n\sqrt{E_g})[(4\alpha_c+10\gamma+20/3)-(g+10\kappa+4\gamma_1)]\{(\mu_1/m)^{3/2}+(\mu_2/m)^{3/2}\} \qquad \ldots \quad (3\,8)$$

in deg/cm-kG, and E_g in eV. As in R we assume $\gamma \cong \gamma_2 \cong \gamma_3$. Also we have replaced $(\boldsymbol{k}\,\boldsymbol{r})^2$ by its average value 1/3. The function $f(\omega/E_g)$ like the function $F_2(\omega/E_g)$ is real below the gap and complex above the gap and when it is complex as before we have to take its real part to calculate the rotation θ. Unlike $F_2(\omega/E_g)$ the form of the real part of the function $f(\omega/E_g)$ is continuous and has a peak at the gap, but again it is not symmetrical about the gap.

Now we consider the contribution due to the transition from the split off valence band V_3 to the conduction band. The basis functions corresponding to V_3 are :

$$\phi_3 = (i/\sqrt{3})|-(X+iY)\beta-Z\alpha|$$

$$\phi_3' = (i/3)|(X-iY)\alpha-Z\beta|.$$

The new basis functions corresponding to V_3 are obtained as before. Calculating the matrix elements with the new basis functions, the contribution to the Faraday rotation θ_4 due to change of matrix element for the transition from V_3 band to the conduction band, according to eq (3 5) is given by,

$$\theta_4 = \theta_{04} H f(\omega/E_g+\Delta) \qquad (3.9)$$

where,

$$\theta_{04} = \left(\frac{4\times 5\,17 E_g f}{n(E_g+\Delta)^{3/2}}\right)(\mu_3/m)^{3/2}$$
$$\times[(\gamma'_1+2\kappa'+\Delta f/3(E_g+\Delta)-(\alpha+4/3)] \qquad \ldots \quad (3.10)$$

in deg/cm-kG, and E in eV. γ'_1 is obtained from γ_1 by replacing f by f'.

Thus eqs (2.3), (3.6), and (3.9) improve upon the approximations (ii) and (iii) discussed in section 1. In the next section we shall try to improve upon the approximation (i) in the long wavelength region.

4 Contribution to Rotation from an Effective Gap

By evaluating the matrix elements only at the Γ point it is tacitly assumed that the contribution to the Faraday rotation near the Γ point energy gap is caused by the transitions at the Γ point band edges. It is true that the contribution to the Faraday rotation due to transitions from other gaps is negligible near any particular gap, but when the frequency range under consideration is away from all the gaps, the transitions at all points in the Brillouin zone are important. Thus it is not surprising that near the low frequency region when it is considerably away from all the energy gaps the rotation under the approximation (i) will deviate from the experimentally observed value. In the calculation of sections 2 and 3 we subtracted off the $\omega = 0$ term to obtain the correct ω^2 dependence but the coefficient of ω^2 term has the wrong magnitude and sign in some cases, so that this device does not properly correct for approximation (i). We therefore re-examine the expression (1.2) for the Faraday rotation at long wavelengths and make use of a different approximation namely that the energy of the incident photon is far enough from all the energy gaps so that the latter can be closely approximated by an average *effective gap*. To begin with we have from eq. (1.2)

$$\sigma_{xy} = \frac{e^2}{im^2} \sum_{nn'} \frac{P^y_{nn'} P^x_{n'n} f_{nn'}}{E_{n_n'} + \omega}$$

This can be written as

$$\sigma_{xy} = \frac{e^2}{im^2} \sum_{n}^{o} \sum_{n}^{u} \frac{P^x_{nn'} P^y_{n'n} - P^y_{nn'} P^x_{n'n}}{E_{nn'^2} - \omega^2}$$

$$+ \frac{e^2}{im^2\omega} \sum_{n}^{o} \sum_{n'}^{u} \frac{E_{nn'}(P^x_{nn'} P^y_{n'n'} + P^y_{n'n} P^x_{n'n})}{E_{nn'^2} - \omega^2}$$

where $\sum_{n}^{o}$ denotes the sum over occupied states, n, and $\sum_{n'}^{u}$ denotes the sum over unoccupied states n'. But for term linear in H we have $\sigma_{xy} = -\sigma_{yx}$. This relation shows that the second term in the above equation is equal to zero So that for terms linear in H we can write

$$\sigma_{xy} = \frac{e^2}{im^2} \sum_{n}^{o} \sum_{n'}^{u} \left[\frac{1}{E^2_{nn'}} + \frac{\omega^2}{E^2_{nn'}(E^2_{nn'} - \omega^2)} \right] (P^x_{nn'} P^y_{n'n} - P^y_{nn'} P^x_{n'n})$$

Since $P^x_{nn'} = imE_{nn} x_{nn'}$, x being the x component of the coordinate operator, the first term becomes zero and

$$\sigma_{xy} = \frac{e^2}{im^2} \omega^2 \sum_{n}^{o} \sum_{n'}^{u} \frac{(P^x_{nn'} P^y_{n'n} - P^y_{nn'} P^x_{n'n})}{E^2_{nn'} (E^2_{nn'} - \omega^2)} \qquad \ldots (4.1)$$

Now we invoke the approximation that the frequency we are interested in is far away from all the gaps of the semiconductor, and that we can replace all the gaps by a single effective gap E_0. Let us write

$$E^2_{nn'} = E_0^2 - (E_0^2 - E^2_{nn'}) = E_0^2 - \Delta E^2_{nn'}$$

where $\Delta E^2_{nn'}$ is the difference in (energy)^2 between the effective gap E_0 and any other real gap $E_{nn'}$. The assumption we make is that $E_0^2 - \omega^2 \gg E_0^2 - E^2_{nn'}$ so that we can write

$$\frac{1}{E^2_{nn'} - \omega^2} = \frac{1}{E_0^2 - \omega^2}\left\{1 - \frac{E^2_{nn'}}{E_0^2 - \omega^2}\right\}^{-1}$$

$$\cong \frac{1}{E_0^2 - \omega^2} + \frac{E_0^2}{(E_0^2 - \omega^2)^2} - \frac{E^2_{nn'}}{(E_0^2 - \omega^2)^2}$$

Substituting this in eq. (4.1) we see that the sum over the first two terms goes to zero as before and the third term gives,

$$\sigma_{xy} = -\frac{e^2\omega^2}{im^2} \cdot \frac{1}{(E_0^2 - \omega^2)^2} \sum_n^o [P^x\ P^y]_{nn}$$

But we have for $\hbar = 1$

$$[P^x, P^y] = eH/ic$$

Therefore,

$$\sum_n^o [P^x, P^y] = eH\mathfrak{n}/ic$$

where $\mathfrak{n}$ is the number of valence electrons per unit volume. Therefore finally we find the Faraday rotation in the low frequency limit to be given by

$$\theta_{l.f.} = \frac{2\pi\mathfrak{n}e^3\omega^2 H}{nc^2m^2(\omega_0^2 - \omega^2)^2} \quad \ldots \quad (4.2)$$

This is the same result as given by classical theory (Moss 1959), which is not surprising as the classical result is also derived on the assumption that the frequency of the incident radiation is away from all absorption edges of the semiconductor. To get the effective gap $E_0 (= \omega_0$ in eV) of a semiconductor with respect to low frequency region we use the expression for refractive index in the low frequency limit as given by classical theory (Moss 1959)

$$n^2_{l.f.} = 1 + \frac{4\pi\mathfrak{n}e^2}{m(\omega_0^2 - \omega^2)} \quad \ldots \quad (4.3)$$

where, n is the number of valence electron per unit volume, e and m are charge and mass of an electron as before. Using the values of the low frequency refrac-

tive index in eq (4.3) we have calculated the values of ω_0, and using these values of ω_0 we have calculated $\theta_{1\cdot f}$ for Ge, GaAs, GaSb, InSb, InAs, and Si. The results of our calculations and the values of the constants used are shown in table 1 with the experimental results For Ge, GaAs, and GaSb we see that

Table 1 Comparison of the theoretical and experimental values of $\theta_{1\cdot f}/\omega^2$ in deg/cm-kG(eV)^2.

Material	Refractive index n	Lattice constant a in Å	Effective gap E_0 in eV	Theoretical value of $\theta_{1\cdot f}/\omega^2$	Experimental value of $\theta_{1\cdot f}/\omega^2$
Ge	4.00	5.66	4.02	3.88	5.74 (77°K & 300°K)
GaAs	3.30	5.66	4.95	2 04	4.35 (77°K) 4.71 (300°K)
GaSb	3.81	6 10	3.79	4 14	5.79 (77°K) 3 44 (300°K)
InSb	3.93	6.49	3.33	5.57	
InAs	3.49	6.07	4.19	3.05	− 48.97 (77°K & 300°K)
Si	3.46	5.43	5.00	2 13	0.22 (300°K)

We have taken the values of refractive index n from Cardona (1965) and lattice constant a from Paul (1961) The experimental values of $\theta_{1\cdot f}/\omega^2$ for Ge, GaAs, GaSb, InSb, InAs and Si are obtained from Walton and Moss (1961), Piller (1964), Piller & Patton (1963), Smith, Pidgeon & Prosser (1962), Cardona (1961), and Piller & Potter (1962) respectively.

the theoretical values compare well with the experimental results, if we consider the insensitivity of the theoretical $\theta_{1\cdot f}$ to the detailed band structure of the material. On the other hand for InSb and InAs the theoretical values are small and positive, the experimental values are large and negative. We attribute this discrepancy to the fact that with respect to the low frequency region the Γ point band gap is not far enough and consequently the limit of the rotation near the band gap predominates. In the case of silicon, the discrepancy between experimental and theoretical results is one order of magnitude, and the cause of this is not yet clear In the next section we examine the relative contributions of the two limiting values

5 The Resultant Rotation and its Comparison with the Experimental Results

Due to the complexity of the problem involved, it is possible only to calculate the rotation in two limiting regions, namely when the energy of the incident radiation is near a particular energy gap and when the energy of the incident

radiation is small compared to all the energy gaps. The former we denote by θ_g and is given by.

$$\theta_g = \theta_1 + \theta_2 + \theta_3 + \theta_4 \qquad \ldots \quad (5.1)$$

where, θ_1, θ_2, θ_3, and θ_4 are given by eqs (1 4), (2.1), (3 6), and (3 9) respectively. And the latter θ_{1f} is given by eq (4 2). To compare the results of our calculations with the experimental results and to see the overall picture we superpose θ_g and θ_{1f}. If we let θ be the resultant rotation arising from the two limiting contributions then,

$$\theta = \lambda\theta_{1f} + \mu\theta_g \qquad (5\,2)$$

where, λ and μ are two variable parameters to be determined by matching the theoretical expression (5 2) with the experimental curve, and give the relative contributions of the two parts For discussion see section 6. We have plotted the theoretical curve according to eq (5 2) by matching λ and μ with the experimental curve for Ge, GaAs, and GaSb at 77°K and 300°K, and is shown in figures 1 to 6 together with the experimental points to facilitate comparison.

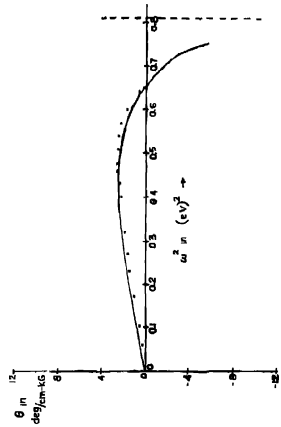

Fig. The Faraday rotation of Ge as a function of ω^2 at 77°K. The solid line is the calculated curve and the points are the experimental points of Walton & Moss (1961). The vertical dashed line is drawn at ω^2 corresponding to E_g at 77°K.

When the spin orbit splitting Δ is large we neglect the contribution from θ_2 and θ_4 which are very small in comparison with θ_1 and θ_3. For InSb and InAs we neglect θ_{1f} and consider only θ since due to the small value of the band gap energy E_g and the large values of the band edge parameters, θ_g predominates

over the whole frequency range and theoretically $\theta_{1\text{-}f}$ is negligible in comparison with θ_g. Thus for materials of very small E_g and large band parameters we assume that θ_g is responsible for the total contributions, or in other words this is the case where the band edge limit is valid. The plot of the theoretical curves

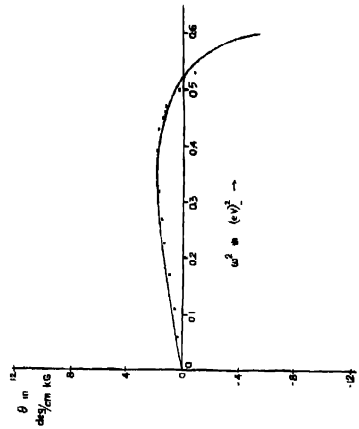

Fig. 2. The Faraday rotation of Ge as a function of ω^2 at 300°K The solid line is the calculated curve and the points are the experimental points of Walton & Moss (1961). The vertical dashed line is drawn at ω^2 corresponding to E_g at 300°K.

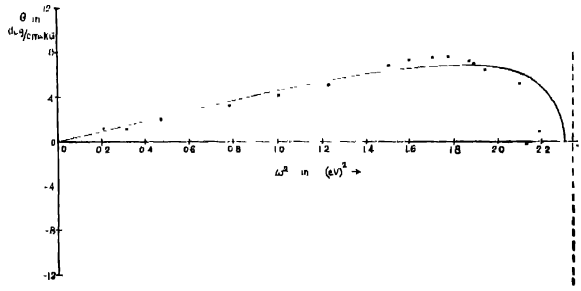

Fig 3. The Faraday rotation of GaAs as a function ω^2 at 77°K. The solid line is the calculated curve and the points are the experimental points of Piller (1964). The vertical dashed line is drawn at ω^2 corresponding to E_g at 77°K

for InSb and InAs at 77°K and 300°K are shown in figures 7 and 8 with the available experimental points. The values of the parameters used in calculating the theoretical curves are shown in table 2. The values of f and h used are the low temperature values since the room temperature values are not known.

For all materials θ, $i = 1$ to 4, are negative and $\theta_{1\cdot f}$ positive. For all the materials θ_3 gives larger contribution than θ_1 near low frequency and θ_1 larger than θ_3 near the band gap. We note, however, that θ_1 and θ_3 are always of the same order of magnitude throughout the whole range we are interested in. For

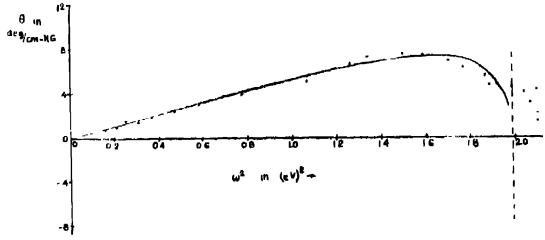

Fig. 4 The Faraday rotation of GaAs as a function of ω^2 at 300°K. The solid line is the calculated curve and the points are the experimental points of Piller (1964). The vertical dashed line as drawn at ω^2 corresponding to E_g at 300°K.

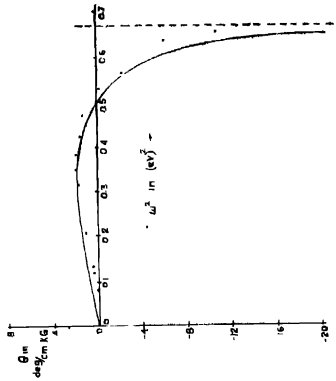

Fig 5. The Faraday rotation of GaSb as a function of ω^2 at 77°K. The solid line is the calculated curve and the points are the experimental points of Piller & Patton (1963). The vertical dashed line is drawn at ω^2 corresponding to E_g at 77°K

Ge, θ_2 is much smaller and is negligible In GaAs, both θ_2 and θ_4 are one order of magnitude smaller than θ_1 and θ_3, and therefore can be neglected. For GaSb, InSb, and InAs due to the large value of the spin-orbit splitting both θ_2 and θ_4 are neglected. Thus in general contributions to θ come mainly from θ_1 and θ_3. From eq. (5.2) we therefore see that the qualitative feature of θ is determined

by the competition between the negative band gap limit rotation θ_g and the positive low frequency limit rotation $\theta_{1\cdot f}$.

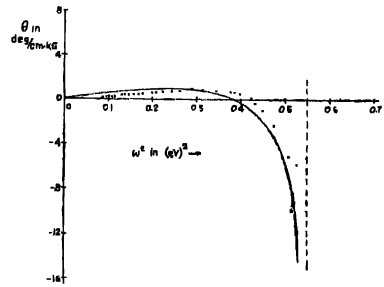

Fig. 6. The Faraday rotation of GaSb as a functio of ω^2 at 300°K. The solid line is the calculated curve and the points are the experimental points of Piller & Patton (1963). The vertical dashed line is drawn at ω^2 corresponding to E_g at 300°K.

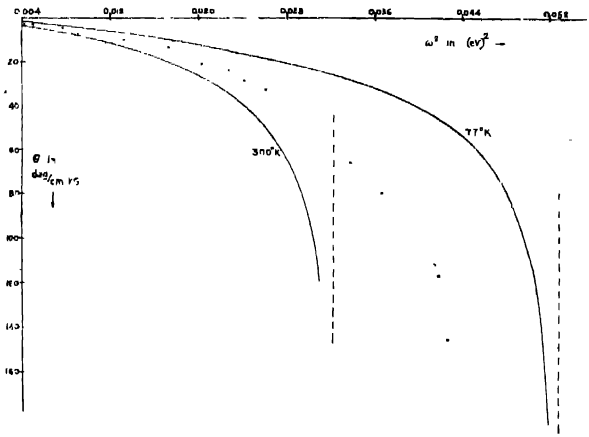

Fig. 7 The Faraday rotation of InSb as a function of ω^2. The solid lines are the calculated curves for two temperatures as indicated; the corresponding energy gaps are indicated by the vertical dashed lines. The points are the experimental points of Smith, Pidgeon & Prosser (1962) at 77°K.

6 Discussion

The results of our calculations as described in the last section show that the quantum mechanical model as developed by Bennett & Stern (1960) and Roth

(1964) provides us with a qualitative and fairly good amount of quantitative understanding of the interband Faraday rotation of the semiconductors we have considered. Our calculations show that the positive rotation at long wavelengths, for Ge, GaAs, and GaSb is due to transitions throughout the entire zone describable in terms of an *effective gap* and not due to the Γ point transitions only as was suggested (Boswarva & Lidiard 1962; Pidgeon *et al* 1964) previously. Calculations in the last section show that the change in the sign of rotation observed in Ge, GaAs, and GaSb as the absorption edge is approached arises from a changing balance between the positive contribution from the *effective*

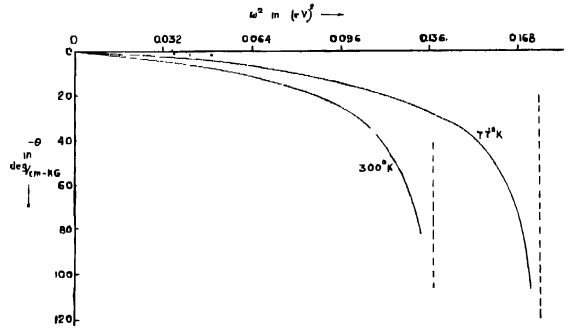

Fig. 8. The Faraday rotation of InAs as function of ω^2. The solid lines are the calculated curves for two temperatures as indicated; the corresponding energy gaps are indicated by the vertical dashed lines. The points are the experimental points of Cardona (1961) which are almost the same for both 77°K and 300°K.

gap and the negative contribution from the gap at the Γ point and not from a changing balance between the contributions from electrons in light valence states and from electrons in heavy valence states as was claimed by Boswarva & Lidiard (1964). It is also important to note here that as was suggested by Lax & Nishina (1961) that the above behaviour is due to a competition between direct transitions at the Γ point and indirect transitions from valence states around the Γ point to the conduction states at the L point, is not tenable, because the strength of the indirect transitions is too weak to give a large enough contribution to the rotation as was shown by Boswarva & Lidiard (1962).

For InSb and InAs due to the reasons mentioned in sections 4 and 5, the Faraday rotations are negative throughout the entire frequency range under consideration. In the case of InSb the present calculation do not make much visible contradiction with the theory developed by Boswarva & Lidiard (1964). But in the case of InAs it does. As according to their calculation the rotation is positive at long wavelengths and changes sign as the gap is approached. The experimental results existing (Cardona 1961) at the present moment for InAs

though scanty seem to support our prediction and therefore also support our explanation of the reversal of sign of rotation in Ge, GaAs and GaSb. A more detailed experimental study of the Faraday rotation in InAs in the frequency range under consideration could be desirable

The theoretical calculations and the experimental results for Ge and GaSb seem to agree quite well throughout the whole frequency range under consideration as is evident from figures 1, 2, 5, and 6 For GaAs the experimental results do not seem to indicate the singularity at the right place, and do not give a smooth curve near the singularity for which it is difficult to compare the theoretical curve with the experimental curve quantitatively near the singularity But the theoretical curve agrees with the experimental curve qualitatively near the singularity and quantitatively near the low frequency region. A remeasurement of the Faraday rotation taking special care of getting the right energy gap could be interesting For exact fit of the theory with the experiment we should expect $\lambda = 1 = \mu$ in eq (5.2) But table 2 shows that for Ge, GaAs, and GaSb, λ is

Table 2. The values of the parameters used to calculate the theoretical curves

Materials	E_g (eV)		Δ(eV)	f	$f+h_1-$	deg/cm-kG				λ		μ	
	77°K	300°K				θ_{01}	θ_{02}	θ_{03}	θ_{04}	77°K	300°K	77°K	300°K
Ge	0.90	0.80	0.29	29	7	-4.2	−4.9	−29.7	−5.6	3.6	3.3	7	.5
GaAs	1.53	1.41	0.33	14	5	−1.4	−0.5	−19.8	−3.7	2.4	2.6	.7	5
GaSb	0.82	0.74	0.86	25	4	−5 3		−34.9		2.6	2 0	.7	.5
InSb	0.23	0.18	0 90	91	10	−16		−124				1	1
InAs	0.41	0.36	0.43	51	6	−12		−63				1	1

The reference for the band parameters for Ge, GaAs, InSb, and InAs are given in Roth (1964). For GaSb we obtain E_g from Piller & Patton (1963), Δ from Ehrenreich (1961). f and f_1h_1 from Cardona (1963)

larger than unity and μ is smaller than unity The fact that λ is greater than unity could be due to the fact that the ω^2 dependent part due to the transition at the Γ point is present both in $\theta_{1\cdot f}$ and in θ_g. The cause for μ is being less than unity could be due to the fact that we evaluate the integral over k from zero to infinity instead of from zero to the length of the Brillouin zone. The smallness of the λ variation with temperature is not surprising because of the insensitivity of $\theta_{1\cdot f}$ to the band parameters and hence to the temperature. The variation of μ with temperature may be due to the fact that we used the low temperature values of the band parameters to calculate θ_{oi}. $i = 1$ to 4, for both temperatures

In the case of InSb the general, form of the experimental curve (Smith, Pidgeon & Prosser 1962) at 77°K is very similar to the theoretical curves at

77°K and 300°K shown in figure 7 and lies between the two theoretical curves The experimental curve at 77°K shows an energy gap which is very different from the gap at 77°K, and seems to correspond to the gap appropriate to a temperature between 77°K and 300°K Because of this difficulty, a detailed quantitative comparison of the two is not possible at this stage. A remeasurement of the Faraday rotation in InSb taking particular care of the band gap could be interesting For InAs detailed experimental result is lacking, the existing experimental result (Cardona 1961) as shown in figure 8 seems to be in good qualitative and quantitative agreement with the theoretical result. As indicated above a detailed experimental measurement could be very interesting and important

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